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***Income inequality in a job-search model with heterogeneous
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Abstract

This paper explores the use of an intertemporal job-search model in the investigation of within-cohort and between-cohort income inequality, the latter being generated by the heterogeneity of time preferences among cohorts of homogenous workers and the former by the cross-sectional turnover in the job market. It also offers an alternative explanation for the empirically-documented negative correlation between time preference and labor income. Under some specific distributions regarding wage offers and time preferences, we show how the within-cohort and between-cohort Gini coefficients of income distribution can be calculated, and how they vary as a function of the parameters of the model.

1 Introduction

Departing from an exogenous distribution of wages faced by a prospective worker, job-search models generate several different derived probability distributions of possible interest for the study of income inequality. This fact,

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though, except maybe for the seminal work of Pissarides's (1974), has not been explored enough in the income-distribution literature.

First, there is the distribution obtained from the original distribution of wage offers by truncation, in which all mass allocated to wages between zero and the reservation wage is placed at zero. Pissarides's use of a job-search model to investigate income inequality, for instance, is wholly based on the range of nonzero incomes of this truncated distribution.

Second, there is the invariant distribution which characterizes the long-run behavior of the state of the system. And, third, the derived distribution of long-run average wages (calculated under the stationary convergent measure) when ex-ante heterogenous workers are grouped into different cohorts¹ of homogenous workers.

In contrast with Pissaride's analysis, our approach is based on the two Markovian long-run distributions last mentioned. Another important conceptual difference between our approach and Pissarides's is that, while this author contemplates solely within-cohort inequality, comparing inequality in two societies with different degrees of risk aversion, we investigate both within-cohort inequality and between-cohort inequality.

In our work, between-group inequality is a consequence of the heterogeneity of time preferences among workers. More impatient workers tend to accept wage offers that less impatient workers possibly would not. The theoretical conjecture driving our investigation, regarding this issue, is that by these means cohorts with more impatient workers end up with lower average wages in a (Markovian) long-run equilibrium, thereby creating between-cohort inequality.

Within cohort-inequality, in turn, is generated by the constant turn over in the job market. At any specific point of time, any cross sectional measurement of incomes in an economy, provided by empirical research, necessarily incorporates the fact that some workers have just been unemployed, others are searching, and others are employed within a certain range of wages. This is the inequality to which we refer as within-cohort inequality. Evaluating it at the stationary distribution amounts to assuming that the economy is in its long-run steady state when incomes are recorded.

It is not a purpose of the paper to provide an explanation of the causes of

¹We shall from now on use the word "cohort" for a group (possibly a society or an economy) with a large number of homogenous workers (or, indistinctly, of homogenous consumers).

income inequality between individuals in different countries or regions. Our main objective, complementing Pissarides's seminal work, is exploring the use of a job-search model in the investigation of income inequality, in particular, by analyzing how the within-cohort and between-cohort inequalities respond to the exogenous parameters of the model².

As a measure of inequality, we shall use only the Gini coefficient of income distribution, and its associated Lorenz curve. It is useful having some idea of the usual empirical values of this statistic. The 75-country sample presented by Bulir (2001) shows that the Gini usually varies between around 0.2 (Czechoslovakia 0.195, Finland 0.202 and Sweden 0.229) and around 0.6 (Brazil leads with 0.633, followed by Gabon, 0.630). Latin America has been one of the regions of the world with the greatest inequality. Ferranti et alii (2004) report Gini coefficients in the 1990s averaging 0.522 in Latin America, against the much smaller figures of 0.342, 0.328 and 0.412, for the OECD, Eastern Europe, and Asia, respectively.

Besides the investigation of the topic of income inequality, the results we derive here can also be used to provide an alternative explanation for the negative correlation between income and time-preference parameters. Lawrance (1991) estimated consumption Euler equations using the Panel Study of Income Dynamics and showed that subjective rates of time preferences can be up to 6 percent higher in the top 5 percent of the income distribution than in the bottom fifth percentile. Two possible explanations of such a pattern have been offered by this author. First, credit constraints (in which case financing the smoothing of consumption during a training period would not be feasible, leading more impatient consumers to lower investments in human capital); and, second, the existence of socioeconomic variables which, following this author, would lead both to time impatience and to a low level of labor income.

Our alternative explanation for Lawrance's empirical findings is that more impatient workers tend to accept lower wage offers and end up with lower average wages. Propositions 1 and 3 of this paper show that such a conjecture is valid under general conditions.

²Other possible ways by means of which impatience can affect inequality, such as its effects on the accumulation of human capital, are not considered here.

2 The Model Without Layoffs

Our theoretical analysis has as exogenous degrees of freedom the distribution of wage offers taken by the workers, the distribution of the time-preference parameter among workers, the probability of layoffs in each period and the average time of compulsory unemployment till another wage offer can be drawn. This last variable can also be interpreted as a required period of retraining. In the present section we do not consider the probability of layoffs. This is done later in the paper.

In the measurable space $([0, 1), \mathcal{B}_{[0,1)}, \mathcal{L})$, standing for the borelians in $[0, 1)$ and \mathcal{L} for the Lebesgue measure in this space, consider a continuum of cohorts of workers. Each cohort j is composed of large number of workers with a (one-period) time preference parameter given by $\beta_j \in [0, 1)$, where

$$\beta_j = H^{-1}(j) \quad (1)$$

In equation (1), j has a uniform distribution in $[0, 1)$ and H stands for the inverse function of a cumulative probability distribution of a random variable taking values in $[0, 1)$, with $H'(\cdot) > 0$ in all of its domain.

The isomorphism (1) allows us to put different probability measures m in the space where the time-preference parameters take value (also $([0, 1), \mathcal{B}_{[0,1)})$). For instance, if H is the cumulative distribution function of a Beta (s, v) random variable, then β_j will be distributed as a Beta (s, v) . Note that having $H'(\cdot) > 0$ allows us to identify each cohort j with its time preference parameter β_j .

In this paper we will endow the space where time preferences take values with two particular distributions: a uniform distribution in $[z, 1)$, $0 < z < 1$ and a Beta (s, v) distribution. By varying s and v one can generate different measures regarding the time preference parameters. In the case of the uniform distribution in $[z, 1)$ the inverse transformation (1) is easily given by:

$$H^{-1}(j) = z + j(1 - z), \quad 0 < z < 1 \quad (2)$$

For $0 < D < \infty$, consider also the second measurable space $(\Omega, \mathcal{B}_{[0,D]}, p)$ and, in this space, the measure q induced by the wage function $w: \Omega \rightarrow [0, D]$. In this induced space $([0, D], \mathcal{B}_{[0,D]}, q)$, we denote by $F(t)$ the distribution function that (q -a.e. -uniquely) determines the measure $q: F(t) = p(w \leq t)$.

Our analysis of the basic decision problem for each worker in each cohort follows, without significant changes, Stokey and Lucas's (1989) version of

McCall's (1970) model of intertemporal job search. A minor modification of our model with respect to McCall's is that we make a distinction between two different periods: the period in which wages are paid (\bar{t}) and the period it takes the unemployed worker to draw a new wage offer (T). Period \bar{t} is also the period with respect to which the time-preference parameter β is defined, and is normalized to unity. Both time periods \bar{t} and T are taken as given by consumers.

By assumption, there are two states regarding the consumer's optimization problem: w and 0. State " w " corresponds to a job offer of w at hand, and state "0" to no job offer. In state w the worker can accept or turn down the offer. If he accepts it, by assumption he stays employed forever with that wage, leading to the present value $w/(1 - \beta)$. If he does not accept the offer he will be this period in state 0. Being in state zero the only thing he can do is wait T periods for a next job offer. During the T periods the worker receives no income (which can be interpreted as a negligible compensation wage). The individual is not allowed to voluntarily quit his job in order to go to the job market again.

Besides this distinction between two time frames, we add to Stokey and Lucas's version of McCall's model by: i) deriving sufficient conditions under which the average wage in the Markovian long-run equilibrium can be proved to be an increasing function of the time-preference parameter (this turns out to be very important in the construction of the between-group Lorenz curves and Gini coefficients); ii) showing that the average wage is a decreasing function of the probability of layoffs and; iii) deriving the associated within-cohort and between-cohort Lorenz curves and Gini coefficients when workers are allowed to have different time preferences according to some arbitrary measure.

The dynamics of the consumer problem is the following: At the beginning of each (unitary) period, each worker in each cohort j (or, given (1), cohort β_j) can choose between two actions: accept a job offer or search for a new wage (w) in T periods. The job offers are drawn from $[0, D]$ according to the measure q . q is known by all workers. The worker is not allowed to borrow or to lend. His consumption c_t is equal to his income w_t in each period.

Consumers in cohort j maximize the expected present value of their consumption:

$$E \left(\sum_{t=0}^{\infty} \beta_j^t c_t \right)$$

With $v(w)$ stating for the value function, the recursive version of the consumer's problem is given by the maximization of:

$$v(w) = \max_{A,R} \left\{ \frac{w}{1 - \beta_j}, \beta_j^T \int_{[0,D]} v(w') dq \right\}$$

To simplify the notation, we make

$$b_j \equiv \beta_j^T \quad (3)$$

The reservation wage is the wage which makes the consumer indifferent between accepting or rejecting the offer. In this version of McCall's model, it is (uniquely) determined by the equation:

$$\bar{w}(b_j) = \frac{1}{1/b_j - 1} \int_{[\bar{w}(b_j), D]} (w - \bar{w}) dq \quad (4)$$

By a direct use of the implicit function theorem or, alternatively, by adding and subtracting $\frac{1}{1/b_j - 1} \int_{[0, \bar{w}(b_j)]} (w - \bar{w}) dq$ to the second term in (4) and integrating $\int_{[0, \bar{w}(b_j)]} (w - \bar{w}) dq$ by parts, it can be easily seen that

$$\bar{w}'(\beta_j) = \bar{w}'(b_j) b_j'(\beta_j) > 0.$$

As shown in Stokey and Lucas (1989), the reservation wage $\bar{w}(j)$ divides $[0, D]$ into two regions: the acceptance region $A(j) = [\bar{w}(j), D]$ and the non-acceptance region $A^c(j) = [0, \bar{w}(j)]$.

Since all employed workers of a certain cohort j have their wages in $A(j)$, for any $I \subset A$ the proportion of wages with $w \in I$ will be given in the limit³ by $q(I)/q(A)$, and the average wage of cohort j , $w_A(j)$, by:

$$w_A(j) = \frac{1}{q(A(j))} \int_{A(j)} w dq \quad (5)$$

³Just note that after a certain number of periods, by the convergence of the $n - th$ order sample statistics to the upper boundary of the distribution (D), all workers in each cohort will end up after a certain number of periods drawing one offer in their respective acceptance region. By the convergence of the empirical distribution to the underlying distribution (Glivenko-Cantelli theorem), measure q emerges naturally. Since only those offers in the acceptance area are relevant for the calculation of the average wage, this must be the region where the integration is performed. Last, a normalization by the q -mass of A is necessary.

For a given measure q , the effect of an increase of j on the average wage $w_A(j)$ is not trivially clear from (5), since the denominator either decreases or remains constant, but so does the integral in the numerator as well (because the integrand is nonnegative and the interval of integration is shortened). It turns out, though, that the division by $q(A(j))$ precisely offsets the fall of the measure of the interval of integration (think of the discrete case). By these means, $w'_A(j) > 0$ simply because the wages averaged are higher when j (and β_j) is higher. We reexamine this issue in Propositions 1 and 3 in the next Sections.

3 The Between-Cohort Income Distribution

The existence of different time preferences between cohorts leads to the between-cohort income inequality. The Lorenz curve plots the percentage of total income earned by the economic agents of a certain economy, when these agents are ordered from those with lower income to those with higher income.

The Gini coefficient (G) is a ratio between two areas. The first area is the one between the the curves $f(j) = j$ and the Lorenz curve $L(j)$. The second area is the one between the curves $f(j) = j$ and $g(j) = 0$. In all cases, j runs from 0 to 1. By integrating:

$$G = 1 - 2 \int_{[0,1)} L(j) dj \quad (6)$$

Suppose, by now, that the income of each cohort j , the Markovian long-run average wage ($w_A(\beta_j)$), is an increasing function of the time preference parameter β_j . Then, by ordering the population by β_j (or equivalently, by j), we are, automatically, also ordering it by income. In this case the Lorenz curve can be easily expressed as a function of j . Indeed, keeping (1) in mind, we can define:

- a) the measure of people with time preference equal or less than β_j : $H(\beta_j) = H(H^{-1}(j)) = j$; note that (by the hypothesis above) this is also equivalent to the measure of people with income less or equal than $w_A(\beta_j)$;
- b) the proportion of income earned by the $j\%$ poorer workers of the economy: $\frac{1}{\bar{w}_A} \int_0^{\beta_j} w_A(\beta_j) dm(\beta_j)$, where $\bar{w}_A = \int_0^1 w_A(\beta_j) dm(\beta_j)$;
- c) the proportion of income earned by the $j\%$ poorer workers of the economy as a function of the population ordered by the size of its income:

given (a) and (b), this is trivially equal to:

$$L(j) = \frac{1}{\int_0^1 w_a(u) du} \int_0^j w_a(u) dm(u) \quad (7)$$

By taking the first and second derivatives, with respect to j , in equation (7) we get $L'(j) = \frac{w_a(j)}{\bar{w}_A}$ and $L''(j) = \frac{w'_a(j)}{\bar{w}_A}$ showing that the Lorenz curve is increasing and (when $w'_a(j) > 0$), strictly convex.

To enhance the intuition about how heterogeneous time preferences can generate income inequality, we exemplify below a calculation of the Gini coefficient of income distribution. The example is developed under the simplest possible (nontrivial) measure q , with wage offers taking only two fixed values in $[0, D]$.

Example 1 *Suppose that the measure q puts mass q_1 in $w_1 \in [0, D]$ and mass q_2 in $w_2 \in [0, D]$, $w_1 < w_2$, with $0 < q_1 < 1, 0 < q_2 < 1$ and $q_1 + q_2 = 1$. Regarding the measure m of the space where time preferences take values, suppose that $H^{-1}(j)$ is given by (2). Making*

$$b_j^* = \frac{w_1}{Ew} \quad (8)$$

and using (4) we get (E is the (unconditional) expectation operator):

$$\bar{w}(b_j) = \begin{cases} b_j Ew, & \text{if } b_j \leq b_j^*, \text{ in which case } \bar{w} < w_1 \\ \frac{b_j w_2 q_2}{1 - b_j q_1}, & \text{if } b_j > b_j^*, \text{ in which case } w_1 < \bar{w} < w_2 \end{cases} \quad (9)$$

Given the assumption about m , time preferences $\beta_j = (b_j)^{1/T}$ across the different cohorts take values in $[z, 1]$. If $(b_{j^})^{1/T} < z$, then all workers in all cohorts will have an average wage w_2 , since for all of them the reservation wage is greater than w_1 (this follows from (9) and (8), which imply that $\bar{w}(\beta_{j^*}) = w_1$, and from the fact that, in this case, $b_j > b_{j^*}$ for all j). Under such conditions the Gini coefficient is equal to zero and there is a perfect income distribution.*

Assume from now the less trivial alternative $0 < z \leq (b_{j^})^{1/T}$. From (2) and (3):*

$$j^* = ((b_{j^*})^{1/T} - z)/(1 - z), \quad 0 \leq j^* < 1 \quad (10)$$

This identification (made through the composition of two strictly increasing functions) allows us rewriting equations (9) above with $j \leq j^*$ and $j > j^*$, respectively, in place of $b_j \leq b_j^*$ and $b_j > b_j^*$. Note that the reservation wage can be situated below w_1 (it is trivially zero when $b_j = 0$) or between w_1 and w_2 , but (also trivially) it cannot exceed w_2 .

Using (5) and (10), the average wage in cohort j is given by:

$$w_A(\beta_j) = \begin{cases} Ew, & j \leq j^* \\ w_2, & j > j^* \end{cases} \quad (11)$$

In this case, since income is a nondecreasing function of j (because $w_2 > Ew$), we can calculate the Lorenz curve integrating directly in j . The Lorenz curve reads:

$$L(j) = \begin{cases} \frac{jEw}{j^*Ew + w_2(1-j^*)}, & 0 \leq j \leq j^* \\ \frac{j^*Ew + w_2(j-j^*)}{j^*Ew + w_2(1-j^*)}, & 0 \leq j^* < j < 1 \end{cases} \quad (12)$$

From (6), upon subdivision of the region of integration into the subintervals from 0 to j^* and from j^* to 1, the Gini coefficient of income distribution is given by:

$$G(w_2, Ew, j^*) = \frac{(w_2 - Ew)j^*(1-j^*)}{j^*Ew + (1-j^*)w_2}, \quad 0 < j^* < 1 \quad (13)$$

The calculation of equation (13) allows for four possible degrees of freedom: the discrepancy of the two wage offers, measured by w_2/w_1 , the probability that the wage drawn from the distribution is equal to w_1 (q_1), the range of time preferences β_j allowed in the economy, $[z, 1)$, which is measured by the parameter z , and the time between job offers T .

The sign of the derivatives of the Gini coefficient with respect to these four parameters depends upon their magnitudes. This follows from (8) and (10) by noticing that

$$j^* = \frac{\left(\frac{w_1}{Ew}\right)^{1/T} - z}{1 - z}$$

and that $G'(j^*)$ is positive when j^* tends to zero and negative when j^* tends to one.

Figure 1 explores some possibilities, regarding these parameters, in the determination of G . In all figures the x -coordinate is q_1 . Along each row, the value of z is kept constant (0 for the first row, 0.75 for the second and 0.9 for the third row) and w_2 assumes the values 1.5, and 5.0, respectively, from the left to the right. Since β_j runs from 0 to 1, and so does β_j^T , the figures can be used with T assuming any value. In all graphs one can observe that there is a value of q_1 strictly between $0 + \delta$ and $1 - \delta$ that maximizes the Gini coefficient; this value seems to increase with w_2/w_1 .

The results displayed in Figure 1 are to be understood under the mechanism of the job search. In this example, all consumers with $j > j^*$ have the same income $w_A = w_2$. They never accept the wage offer w_1 . Therefore, the reason generating the concentration of income is the fact that consumers with $j \leq j^*$ do accept w_1 when it is offered, ending up with an average income $Ew < w_2$. If w_1 is never drawn ($q_1 = 0$), or is always drawn ($q_2 = 1$), or if all time preference parameters are located on or above the cutoff point $(b_j^*)^{1/T}$, or if the two wage offers are equal, there is no concentration of income.

The fact that came up in the example above, that less impatient workers, as measured by β_j , end up with higher average wages, is proved (with $\theta = 0$) in Proposition 1 below. Since $w'_A(j) = w'_A(\beta_j)\beta'_j(j)$ and since $\beta'_j(j)$ is always trivially positive (by (1)), $w'_A(\beta_j) > 0$ implies $w'_A(j) > 0$. This means that in this case j orders the consumers from the poorer to the richer, thereby allowing (as we did in the particular case above) the simplified construction of the Lorenz curve denoted by (7).

Proposition 1 *The average wage w_A is an increasing function of the time preference parameter β_j .*

Proof. Case1- If Remark 1 in Section 4 is valid and $F(w)$ has a density function, just make $\theta = 0$ in the proof of Proposition 3 of the next Section.

Case 2 - If q has a discrete distribution with masses $q_1, \dots, q_{a_1-1}, q_{a_1}, \dots, q_{a_x}, \dots, q_{a_n}$, then the Lebesgue integral $\frac{1}{q(A(j))} \int_{A(j)} w dq$ reads $\frac{\sum_{i=a_1}^{a_n} w_i q_i}{\sum_{i=a_1}^n q_i}$ where the integer a_1 satisfies $w_{a_1} \geq \bar{w} \geq w_{a_1-1}$. Suppose that the wages are indexed in nondecreasing order:

$$w_1 \leq \dots \leq w_{a_1} \leq w_{a_2} \leq \dots w_{a_x} \leq \dots \leq w_{a_n} \quad (14)$$

When j increases by $\Delta \bar{w}$, either it still happens that $w_{a_1} \geq \bar{w} + \Delta \bar{w} \geq w_{a_1-1}$, in which case the average wage remains constant, or that, for $n > x > 1$,

$w_{a_x} \geq \bar{w} + \Delta\bar{w} \geq w_{a_x-1} \geq w_{a_1}$. In the last case, the average wage either increases or remains constant, due to the general fact that, given (14), for any $x > 1$:

$$\frac{\sum_{i=a_1}^{a_n} w_i q_i}{\sum_{i=a_1}^n q_i} \leq \frac{\sum_{i=a_x}^{a_n} w_i q_i}{\sum_{i=a_x}^n q_i}$$

Indeed, this is equivalent to having:

$$(q_{a_x} + \dots + q_{a_n}) \sum_{i=a_1}^{i=a_x-1} q_i w_i \leq (q_{a_1} + \dots + q_{a_x-1}) \sum_{i=a_x}^{i=a_n} q_i w_i$$

which is true by (14). ■

4 The General Model

In this section we introduce the possibility that, once employed, any worker can be laid off, in the beginning of each period, with a fixed and known probability θ . The value function now reads:

$$v(w) = \max_{A,R} \left\{ w + (1 - \theta)\beta_j v(w) + \theta b_j \int_{[0,D]} v(w') dq, \quad b_j \int_{[0,D]} v(w') dq \right\}$$

and the reservation wage is given by:

$$\bar{w}(j) = \frac{b_j}{1 - b_j + \theta k} \int_{[\bar{w}(j), D]} (w - \bar{w}) dq \quad (15)$$

where

$$k = \beta_j \frac{1 - b_j}{1 - \beta_j}.$$

Proposition 2 *The reservation wage is an increasing function of the time-preference parameter β_j .*

Proof. We want to show that:

$$\bar{w}'(\beta_j) > 0 \quad (16)$$

Add and subtract $\frac{b_j}{1-b_j+\theta k} \int_{[0, \bar{w}(j)]} (w - \bar{w}) dq$ to the second member of (15) and use (note that $dF = dq$ here)

$$\int_{[0, \bar{w}(j)]} (w - \bar{w}) dF(w) = \int_{[0, \bar{w}(j)]} F(w) dw$$

to get:

$$\bar{w}(\beta_j) = \frac{b_j}{1 + \theta k} \left[Ew + \int_{[0, \bar{w}(j)]} F(w) dw \right]$$

Suffices, therefore, showing that the term $\frac{b_j}{1+\theta k}$ is an increasing function of β_j . Note that:

$$k = \beta_j + \beta_j^2 + \dots + \beta_j^T \quad (17)$$

Then:

$$\frac{b_j}{1 + \theta k} = \frac{\beta_j^T}{1 + \theta(\beta_j + \beta_j^2 + \dots + \beta_j^T)} = \frac{1}{\beta_j^{-T} + \theta(\beta_j^{1-T} + \beta_j^{2-T} + \dots + 1)}$$

The denominator is a sum of functions each one of which has a negative derivative with respect to β_j . The result follows trivially. ■

As shown in Stokey and Lucas (1989), from which we borrow the analysis below, introducing the possibility of layoffs leads to a more interesting dynamics. Remember the sets A and A^c defined before. The rules of the optimization by the worker define a transition function $P : [0, D] \times \mathcal{B}_D \rightarrow [0, 1]$. For an unemployed worker ($w \in A^c$), the probability of having an offer in any borelian $B \subset [0, D]$ is given by $q(B)$. A worker employed with wage w (in which case, necessarily, $w \in A$) can only lose his job (with probability θ) or keep the same wage next period. Therefore, with probability zero he will have a wage in a borelian B that does not contain either 0 or w . If the borelian B contains 0, but not w , or w but not zero, the transition probabilities are, respectively, θ and $1 - \theta$. If it contains both, since these are disjoint events (because $0 \notin A$), $P(w, B) = 1$.

We now introduce a new measure λ_t in $([0, D], \mathcal{B}_{[0, D]})$, representing the (wage) state of workers of a certain cohort j (the j is omitted), in period t .

Associated with the transition function P is the operator T^* which acts on any probability measures in the space $([0, D], \mathcal{B}_{[0, D]})$, λ in particular. This operator is defined, for all $B \in \mathcal{B}_{[0, D]}$, by:

$$(T^* \lambda)(B) = \int P(w, B) \lambda(dw)$$

We are interested in calculating the average wage (for a fixed j) of different cohorts of the economy in the long run. Therefore, for our purposes it is important to know the limiting measure of the state of the economy, particularly for sets $C \subset A$ (of employed workers). A worker of a cohort j , in period $t + 1$, is employed with wage $w \in C$, if and only if he was unemployed and got a wage offer $w \in C$ or he was already employed with wage w in the beginning of period $t + 1$ and kept his job. Given the assumed independence of job offers in each period, we can write

$$\lambda_{t+1}(C) = \lambda_t(A^c)q(C) + \lambda_t(C)(1 - \theta) \quad (18)$$

The determination of the long-run measure $\lambda(C) = \lim_{t \rightarrow \infty} \lambda_t(C)$ requires the calculation of $\lambda_t(A^c)$. Since a worker can only be unemployed in period $t + 1$ iff he was already unemployed and drew a wage offer in A^c or was employed and lost his job, we have:

$$\lambda_{t+1}(A^c) = \lambda_t(A^c)q(A^c) + \lambda_t(A)\theta \quad (19a)$$

Taking limits, equation (19a) trivially implies $\lambda(A^c) = \theta/(\theta + q(A))$. Taking limits in (18) and using this result yields, for $C \subset A$:

$$\lambda(C) = \lim_{t \rightarrow \infty} \lambda_t(C) = \frac{q(C)}{\theta + q(A)}$$

We are now ready to calculate the average wage in each cohort j of the economy. This is given by:

$$w_A(j) = \int_{[\bar{w}(j), D]} \frac{wdq}{\theta + q(A(j))} \quad (20)$$

where $\bar{w}(j)$ follows (15). As before, it is not clear at a first glance if the average wage is an increasing function of the time preference parameter. We prove it below for the continuous case.

Remark 1 ⁴ $F'(w)$ exists and is absolutely continuous a.e. in $[0, B]$, with $F'(w) \equiv f(w) > 0$ for (q - a.e) all w in $[0, D]$

Proposition 3 Under Remark 1, the average wage is an increasing function of the time preference parameter β_j .

⁴Remember the definition of $F(w)$ as the distribution function determined by q .

Proof. Using the distribution function F and its respective density function with respect to the Lebesgue measure in \mathbb{R} , it follows from the result above that the average wage is now given by:

$$w_A(\beta_j) = \frac{1}{\theta + 1 - F(\bar{w}(\beta_j))} \int_{[\bar{w}(\beta_j), D]} w f(w) dw \quad (21)$$

Taking the derivative with respect to β_j :

$$w'_A(\beta_j) = \frac{\partial w_A}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \beta_j} = \frac{f(\bar{w}(\beta_j))(\int_{[\bar{w}(\beta_j), D]} w - \bar{w}(\beta_j) dq - \bar{w}\theta)}{(\theta + 1 - F(\bar{w}(\beta_j)))^2} \frac{\partial \bar{w}}{\partial \beta_j}$$

Since $\frac{\partial \bar{w}}{\partial \beta_j} > 0$ by (4), suffices showing that $\int_{[\bar{w}(\beta_j), D]} (w - \bar{w}(\beta_j)) dq - \bar{w}\theta$ is an increasing function of β_j . Write:

$$\int_{[\bar{w}(\beta_j), D]} (w - \bar{w}(\beta_j)) dq - \bar{w}\theta = \bar{w} \frac{1 - b_j + \theta(k_j - b_j)}{b_j}$$

and use (17) to get:

$$\int_{[\bar{w}(\beta_j), D]} (w - \bar{w}(\beta_j)) dq - \bar{w}\theta = \bar{w} \frac{1 - b + \theta(\beta_j + \beta_j^2 + \dots + \beta_j^{T-1})}{b_j} > 0$$

■

The demonstration uses the fact that the probability of layoff does not depend upon the reservation wage.

Although we have used Remark 1 to prove Proposition 3 (though not Proposition 1), this is not a necessary condition. Indeed, we extend Example 1 below to the case when $\theta \neq 0$ (with $T = 1$) and show that, once more, the long-run average wage increases with the time preference parameter.

Example 2 *This example introduces a probability of layoff θ in Example 1. We take T to be equal to one (in which case $\beta_j = b_j = k_j$). Define:*

$$\beta_j^{**} = \frac{w_1}{Ew - w_1\theta} \quad (22)$$

The reservation wage reads:

$$\bar{w}(\beta_j) = \begin{cases} \frac{b_j}{1+b_j\theta}Ew, & \text{if } \beta_j \leq \beta_j^{**}, \text{ in which case } \bar{w} \leq w_1 \\ \frac{b_j w_2 q_2}{1-b_j(1-\theta-q_2)}, & \text{if } \beta_j > \beta_j^{**}, \text{ in which case } w_1 < \bar{w} < w_2 \end{cases}$$

and the average wage:

$$w_A(\beta_j) = \begin{cases} \frac{Ew}{\theta+1}, & \text{if } \beta_j \leq \beta_j^{**} \\ \frac{w_2 q_2}{\theta+q_2}, & \text{if } \beta_j > \beta_j^{**} \end{cases}$$

The existence of the two average wages requires, in (22), $0 \leq \beta_j^{**} \leq 1$, which implies

$$w_2 \geq w_1 \left(1 + \frac{\theta}{q_2}\right) \quad (23)$$

A nondecreasing average wage, as a function of β_j requires

$$\frac{w_2 q_2}{\theta + q_2} \geq \frac{Ew}{\theta + 1}$$

which is ensured by (23).

Another possibility we want to check is if the intuition that the average wage should vary negatively with the probability of unemployment does in fact hold. The result does not follow at a first glance from (21). θ affects the average wage both directly, through the term outside the integral, and indirectly, through the reservation wage. We know from (15) that, as θ increases, the reservation wage falls. Since F is a nondecreasing function with values no greater than one, $\theta + 1 - F(\bar{w}(\beta_j))$ increases and the term outside the integral decreases. The effect of θ on the integral, though, is positive. Increasing θ decreases the reservation wage and, since the integrand is positive, increases the integral. The final result is then a combination of two effects of opposite sign. Proposition 4 uses a result in the proof of Proposition 3 to show that the negative effect outweighs the positive one:

Proposition 4 *The average wage is a decreasing function of the probability of unemployment θ .*

Proof.

$$\frac{\partial w_A}{\partial \theta} = \frac{\partial w_A}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \theta} + \frac{\partial w_A}{\partial \theta}$$

From the demonstration of Proposition 3, $\frac{\partial w_A}{\partial \bar{w}} > 0$ and therefore the first term of the second member above is negative. The second term is also trivially negative, as one can observe from (21). ■

5 The Within-Cohort Income Distribution

In this Section we consider the fact that, within each cohort, a cross-sectional analysis of income will find heterogeneous situations among consumers. Some have just been laid off, others have just turned down a wage offer, while others are employed with different wages, ranging, possibly, from the reservation wage to the wage at the top of the distribution (here, D). The calculation of the within-cohort inequality is based on the assumption that each cohort has a very large number of workers. By the result in probability theory that the empirical distribution converges almost surely to the actual distribution (Glivenko-Cantelli theorem), this allows us to use the Markovian stationary distribution as the distribution of incomes in the cohort.

The tools to measure the within-cohort income inequality are essentially the same as those presented in Section 3. Proceeding with the calculations, though, necessarily requires the specification of a original measure over wage offers, q . In what follows, we assume that q is given by the Lebesgue measure in $[0, 1]$ (in which case $q(A) = 1 - \bar{w}$). Using the convergence results of section 4:

$$\lambda(w) = \left\{ \begin{array}{ll} \frac{\theta}{\theta+1-\bar{w}}, & w = 0 \\ 0, & 0 < w < \bar{w} \\ \frac{1}{\theta+1-\bar{w}}, & \bar{w} \leq w \end{array} \right\}$$

The Lorenz curve as a function of w is given by:

$$L(w) = \left\{ \begin{array}{ll} 0, & 0 \leq w < \bar{w} \\ \frac{w^2 - \bar{w}^2}{1 - \bar{w}^2}, & \bar{w} \leq w \leq 1 \end{array} \right\}$$

The cumulative distribution function associated with the stationary measure λ is:

$$S(w) = \begin{cases} \frac{\theta}{\theta+1-\bar{w}}, & 0 \leq w < \bar{w} \\ \frac{\theta}{\theta+1-\bar{w}} + \int_{\bar{w}}^w \frac{1}{\theta+1-\bar{w}} du = \frac{\theta+w-\bar{w}}{\theta+1-\bar{w}}, & \bar{w} \leq w \leq 1 \end{cases}$$

in which case the Lorenz curve as a function of D reads:

$$L(S) = \begin{cases} 0, & 0 \leq S < \frac{\theta}{\theta+1-\bar{w}} \\ \frac{(S(\theta+1-\bar{w})+\bar{w}-\theta)^2-\bar{w}^2}{1-\bar{w}^2}, & \frac{\theta}{\theta+1-\bar{w}} \leq S \leq 1 \end{cases} \quad (24)$$

Make:

$$U = \int_0^1 L(S) dS = \int_{\frac{\theta}{\theta+1-\bar{w}}}^1 \frac{(S(\theta+1-\bar{w})+\bar{w}-\theta)^2-\bar{w}^2}{1-\bar{w}^2} dS$$

By a second change of variable, if $u = S(\theta+1-\bar{w})+\bar{w}-\theta$, the above integral reads:

$$U = \int_{\bar{w}}^1 \frac{u^2 - \bar{w}^2}{(1-\bar{w}^2)(\theta+1-\bar{w})} du$$

$$U = \frac{1 - 3\bar{w}^2 + 2\bar{w}^3}{3(1-\bar{w}^2)(\theta+1-\bar{w})}$$

which by (6) leads to the within-cohort Gini coefficient:

$$G = \frac{1 + 3\theta - \bar{w}(\bar{w}^2 - 3\bar{w}(1-\theta) + 3)}{3(1-\bar{w}^2)(\theta+1-\bar{w})} \quad (25)$$

where \bar{w} is determined by (15) by making q the Lebesgue measure in $[0, 1]$.

- Averaging the Within-Cohort Gini Across Cohorts

In order to have an idea of how the within-cohort Gini compares with the between-cohort coefficient we need to average it out across all different cohorts. Pyatt (1976) (see also Yao (1999)) has devised a methodology to make such aggregation. Roughly speaking, the within-index of each cohort is weighed by the average wage of the respective cohort and averaged out by the prevailing distribution of cohorts in the economy. We follow such a procedure in the calculations of example 3 below. Since the average wage is an increasing function of the time preference, this methodology implies that within-Gini coefficient of cohorts with higher time preference will be more important for the mean across cohorts than those with lower time preferences.

- No Additivity

Note that the Gini coefficient is not an additive measure of income inequality. Therefore, one cannot add the between-cohort (G_B) and the within-cohort (G_W) coefficients in order to get a total Gini. However, as it has been shown by Pyatt (1976), a decomposition of the type $G = G_B + G_W + G_O$ is possible, with G_O standing for a correction due to the overlapping of incomes. G_O is zero when the income of the different groups do not overlap. In our case, such overlapping clearly happens, since individuals from cohorts with higher average income (higher time-preference parameter) will sometimes be unemployed with a wage equal to zero, thereby being, temporarily, in a worse situation than other individuals from lower income cohorts.

The important point to notice, though, is that G_O has been shown by Pyatt to be always nonnegative. For this reason, in example 3 below we refer to $G_B + G_W$ as a lower bound (LB) to the overall Gini coefficient of the economy.

6 Quantitative Considerations

Our next step will be devising a numerical example that can help us getting some quantitative insight into the problem. In order to do so, we consider a uniform distribution for the wage offers and a Beta distribution for the time preference parameters.

Example 3 *In an economy where in each period the workers face a probability θ of layoff, suppose that q is characterized by a uniform distribution in*

$[0, 1]$. Using (15) we get:

$$\bar{w}(k_j(\beta_j), b_j(\beta_j)) = \bar{w}(\beta_j) = \left[\frac{1 + k_j\theta}{b_j} - \sqrt{\left(\frac{1 + k_j\theta}{b_j}\right)^2 - 1} \right] \quad (26)$$

By (20), the average wage reads:

$$w_A(\beta_j) = \frac{1 - \bar{w}(\beta_j)^2}{2[(\theta + 1) - (\bar{w}(\beta_j))]} \quad (27)$$

Figures 2 and 3 show how the reservation wage changes as a function of the time preference and the probability of layoff, the first varying between zero and one, and the second assuming the (extreme) values 0, 0.25, 0.50 and 0.75. In Figure 2, $T = 1$ and, in Figure 3, $T = 6$. Figures 4 and 5 repeat the same procedure regarding the average wage.

The within-cohort income inequality has already been calculated when q is the $[0, 1]$ uniform distribution and is given by (24) and (25).

We now proceed with the calculation of the between-cohort Lorenz curve $L_B(k)$, $0 \leq k \leq 1$, and Gini coefficient G_B . Using (7), (26) and (27), as well as the expressions of k_j , b_j and β_j as functions of j :

$$L_B(k) = \frac{1}{\bar{w}_A} \int_0^k \frac{1 - \left[\frac{1 + k_j\theta}{b_j} - \sqrt{\left(\frac{1 + k_j\theta}{b_j}\right)^2 - 1} \right]^2}{2 \left[(\theta + 1) - \left[\frac{1 + k_j\theta}{b_j} + \sqrt{\left(\frac{1 + k_j\theta}{b_j}\right)^2 - 1} \right] \right]} dj$$

where

$$\bar{w}_A = \int_0^1 \frac{1^2 - \left[1 - \left[\frac{1 + k_j\theta}{b_j} - \sqrt{\left(\frac{1 + k_j\theta}{b_j}\right)^2 - 1} \right] \right]^2}{2 \left[(\theta + 1) - \frac{1 + k_j\theta}{b_j} + \sqrt{\left(\frac{1 + k_j\theta}{b_j}\right)^2 - 1} \right]} dj$$

From which the Gini coefficient is given by (6), with

$$U = \int_0^1 \int_0^k \frac{1 - \left[\frac{1 + k_j\theta}{b_j} - \sqrt{\left(\frac{1 + k_j\theta}{b_j}\right)^2 - 1} \right]^2}{2 \left[(\theta + 1) - \frac{1 + k_j\theta}{b_j} + \sqrt{\left(\frac{1 + k_j\theta}{b_j}\right)^2 - 1} \right]} dj dk$$

In the first simulation below m is the measure of a Beta (114.5, 1.01) distribution function. The parameters have been chosen in order to make the

(monthly) average equal to $114.5/(114.5 + 1.01) = 0.9913$. This corresponds to an average yearly time-preference of 0.900. 95% of the mass of this distribution is concentrated on the (yearly) interval (0.6779, 0.9972).

The between-cohort and within-cohort Lorenz curves are shown, respectively, in Figure 6 and 7. Equally, the values of both the Gini coefficients, for different values of T and θ , are shown in Table 1 below:

Table 1
*Within-Cohort (G_W), Between Cohort (G_B) and
Lower Bound (LB) Gini Coefficients as a Function
of the Number of Periods to get a Job Offer (T)
and the Probability of Unemployment (θ)*

	$T = 6$			$T = 12$		
	G_W	G_B	LB	G_W	G_B	LB
$\theta = 0$	0.076	0.031	0.107	0.106	0.042	0.148
$\theta = 1/120$	0.110	0.018	0.128	0.141	0.024	0.165
$\theta = 10/120$	0.258	0.005	0.263	0.288	0.007	0.295

The remainder of this section is based on the data generated by the example above.

- The Effect of Theta

Note in Table 1 (see also Figure 6) that the between-cohort inequality decreases when the probability of layoff increases, whereas the opposite happens with the within-cohort inequality. Figures 2-5 can help us to understand the fall of the between-cohort inequality. An increase in θ has two effects. First, it decreases the reservation wages for the different cohorts (Figures 2 and 3), thereby making it more likely that low wage offers are taken by workers in different cohorts. Second, by being unemployed more frequently, the average wage of workers in different cohorts decreases (Figures 4 and 5), impoverishing all workers at the same time and decreasing inequality.

The within-cohort inequality increases (see Figures 7 and 8) when θ increases, for, at first, two reasons. First, because more mass of the stationary

distribution is concentrated at the point of zero income, an increasing function of θ (remember that the total mass of this point is equal to $\theta/(\theta+1-\bar{w})$). Second, at the same time, the range of nonzero wages increases due to the fall (when θ increases) of the reservation wage⁵.

- The Effect of T

Table 1 above shows that by increasing the time period in which new job offers are taken one increases the between-cohort inequality (see Figure 6). Longer planning horizons magnify the (usually tiny) discrepancies of time preference among agents. This point is important when one considers between-jobs training, an activity that can demand discrete time horizons taking several years.

With respect to the (averaged) within-cohort Gini, Table 1 and Figure 9 (in which $\theta = 0$) seem to lead to the same positive relation. However, by increasing the interval of variation of T and making $\theta = 10/120$, as shown in Figure 10, we conclude the positive correlation between GW and T does not necessarily hold.

- A Comparison with Available Empirical Data

In our formulation of the job-search problem, being employed or not in the next period is a Bernoulli random variable. Let us call “success” the event of being laid off next period. Then the number of periods till the first “success” occurs is a geometric random variable, with average $1/\theta$. If we define the period in which wages are paid (\bar{t}) as monthly, $\theta = 1/120$ translates, in average, one layoff each ten years. For the purpose of our quantitative assessments of the problem, regarding a comparison with real-world numbers, we concentrate on this case. Regarding the time-preference parameter, we consider the Beta distribution with an yearly average of 0.9 and a compulsory time to get a new wage offer of six months.

Such choices lead us to the 1 by 3 submatrice (2, 2) in the 3×2 matrix of Table 1, where one reads within-cohort and between-cohort Gini coefficients of, respectively, 0.110 and 0.018. Comparing the lower bound $LB = 0.128$ to the numbers mentioned in the introduction, this is around 60% of the order of magnitude of the Gini coefficients of Czechoslovakia, Finland and

⁵The fact that an increase of the nonzero range of the truncated distribution leads to an increase in inequality has been advocated by Pissarides (1974).

Sweden, countries where income inequality is very low, or around 20% of that of Brazil or Gabon, countries located on the other side of the distribution of Gini coefficients. As one can notice from element 3×1 in the table, though, the Gini coefficients of the model increase significantly when one allows for higher probabilities of unemployment.

- Restrictions

Note that all the numerical results above, upon which we have studied the model, were based on two arbitrary probability distributions: a Beta distribution for the time preferences, and a uniform distribution for the wage offers. Therefore, the results should be interpreted under this proviso.

7 Conclusions

In this paper we have built on Stokey and Lucas's (1989) version of McCall's (1970) model, in order to explore the use of a job-search model in the investigation of income inequality. The model leads to both a within-cohort income inequality, as found in empirical cross-sectional analyses of income data, and to a between-cohort income inequality, due to the fact that workers are allowed to have heterogenous time preferences.

Our analysis had as exogenous degrees of freedom the distribution of the wage offers taken by the workers, the distribution of the time preference parameter among consumers, the probability of layoffs in each period and the number of periods it takes for a new job offer to be drawn. In applied work, such distributions should be estimated from the available data.

From a theoretical perspective, we have adapted Stokey and Lucas' version of McCall's model by allowing the time period in which wages are paid and the time period in which new wage offers are made to differ. We have also derived sufficient conditions under which the average wage which emerges from the long-run Markovian equilibrium can be proved to be a nondecreasing function of the time preference. This point is important in the construction of the between-cohort Lorenz curves. Third, we have shown how the within-cohort and the between-cohort Gini coefficients can be calculated under different distributions of wage offers and time preferences.

A quantitative idea of the problem was made available through the analysis of three examples. The first two examples were based on a two-point distribution for the wage offers and on a uniform distribution for the time

preferences. The third example dealt with a uniform distribution of wage offers and a Beta distribution of time preferences.

Examples 1 and 2 allowed us to have a measure of how the concentration of income tends to worsen when the disparity among the different wage offers increase. We have also seen that the two-point distribution of wages allows for a wide range of the between-cohort Gini coefficient, depending upon the parameters of the problem.

From example 3 we have learned, regarding the within-cohort inequality, that both an increase in the rate of layoffs or an increase in the time between job offers (T) tends to increase it. Regarding the between-cohort inequality, we have learned that (in contrast to the within-cohort inequality), it tends to decrease with the increase of the probability of unemployment, and (in agreement with the within-cohort inequality) to increase with the time between job offers. Besides, as one would expect, the greater the range of time preferences among economic agents, the greater the resulting long-run between-cohort inequality⁶. We have also learned that the between-cohort Gini coefficients that can be generated solely in terms of heterogeneous-time-preferences, under an intertemporal job-search rationale, are of a very low order of magnitude.

Such considerations suggest a line for further research. A natural extension of the framework developed here is explicitly modelling the time interval T as a training period and/or by making it random. The main difference with respect to the present analysis is that each time the worker would accumulate training, he would draw wages from a more favorable distribution. The determination of this new probability distribution should then take into consideration the fact that, the higher the (accumulated) training period, the higher should be the chances of drawing better wages, thereby favoring patient workers.

⁶This can be concluded by considering, for the time preferences, beta distributions with more dispersed mass or uniform distributions in $[z,1)$, with z assuming different values in $[0,1)$.

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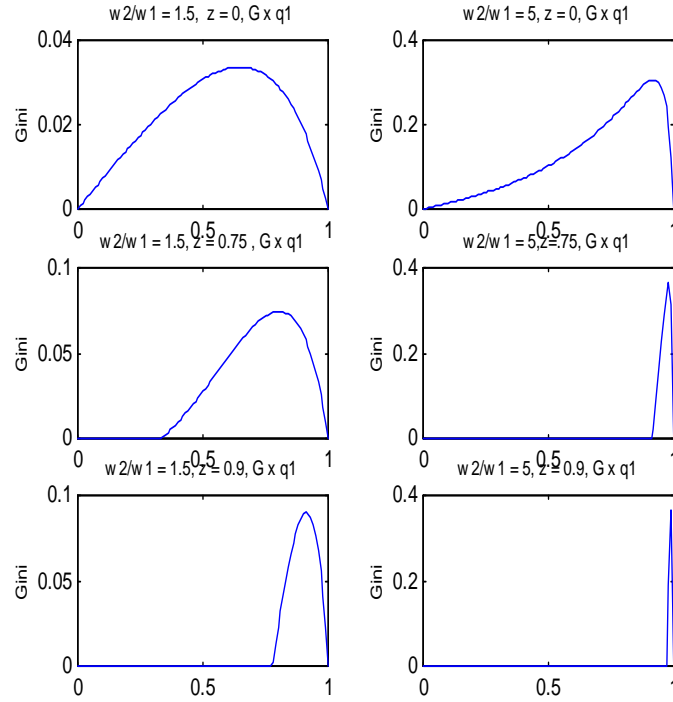


Figure 1: (Example 1) - Reservation Wage, Average Wage and Gini Coefficients as a Function of q_1 (in the x-axis), the Range of Time-Preference Parameters (from the ceiling to the bottom) and w_2/w_1 (from the right to the left).

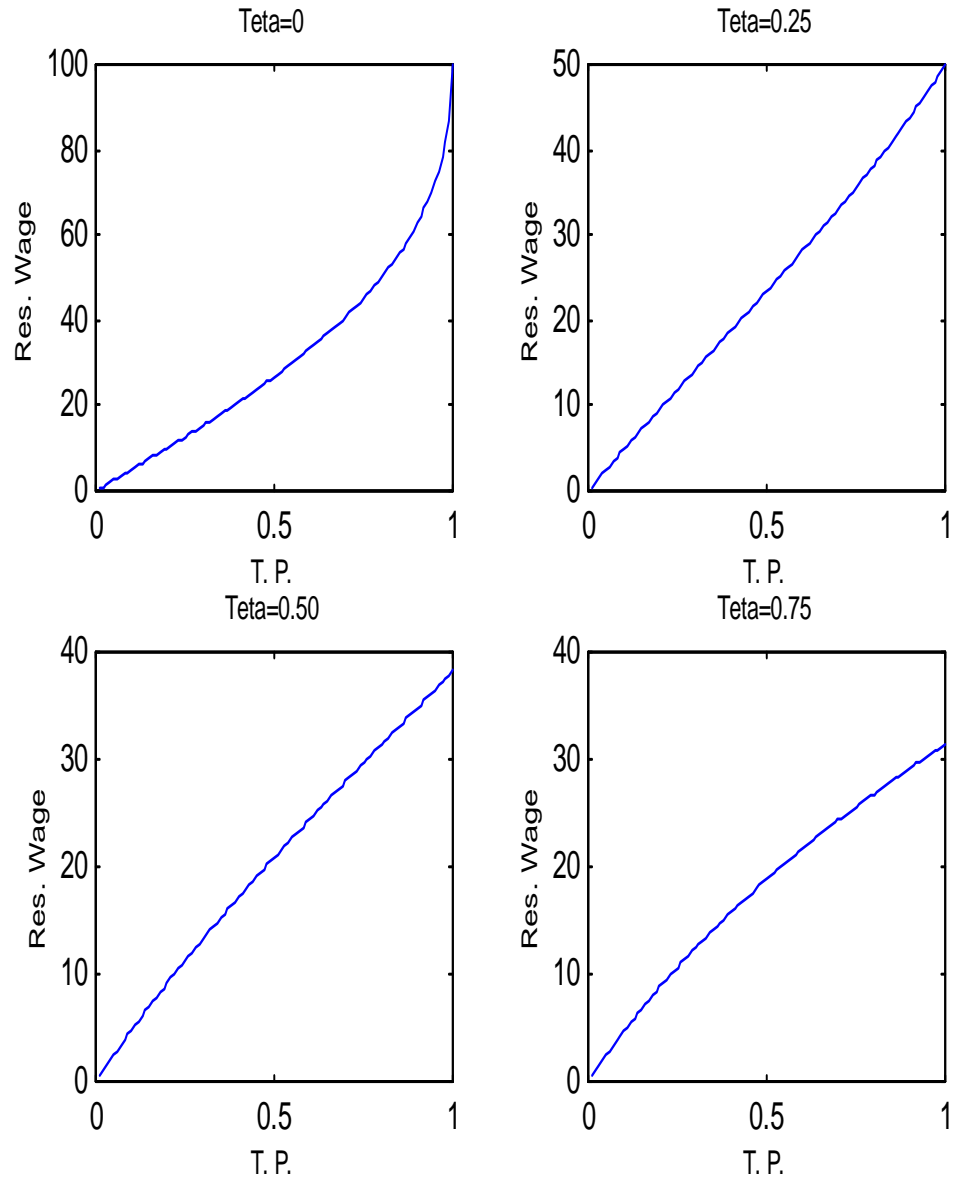


Figure 2: (Example 2)- Reservation Wage as a Function of Time Preference for Different Values of Theta and $T=1$.

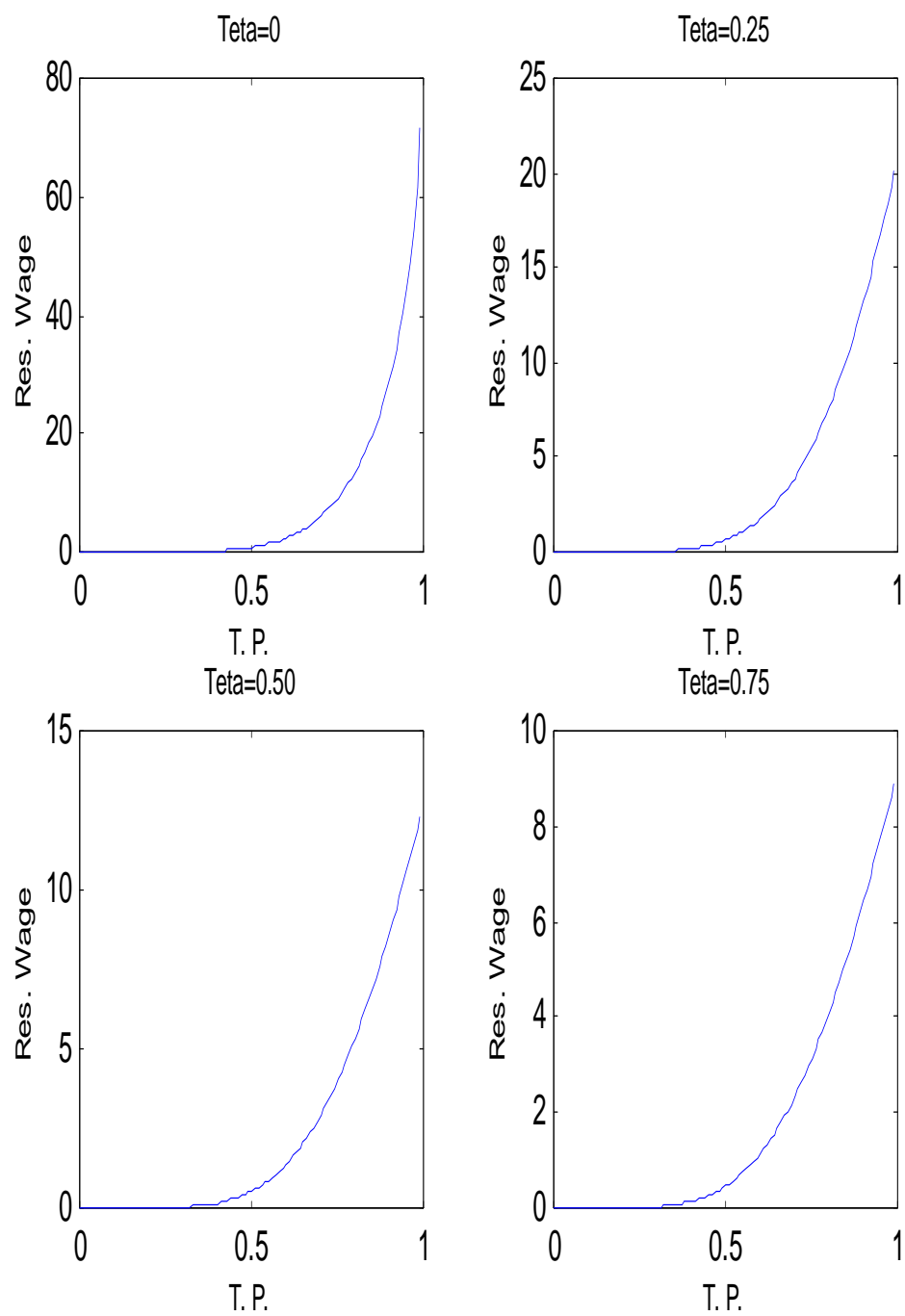


Figure 3: (Example 2) - Reservation Wage as a Function of Time Preference for Different Values of Θ and $T=6$.

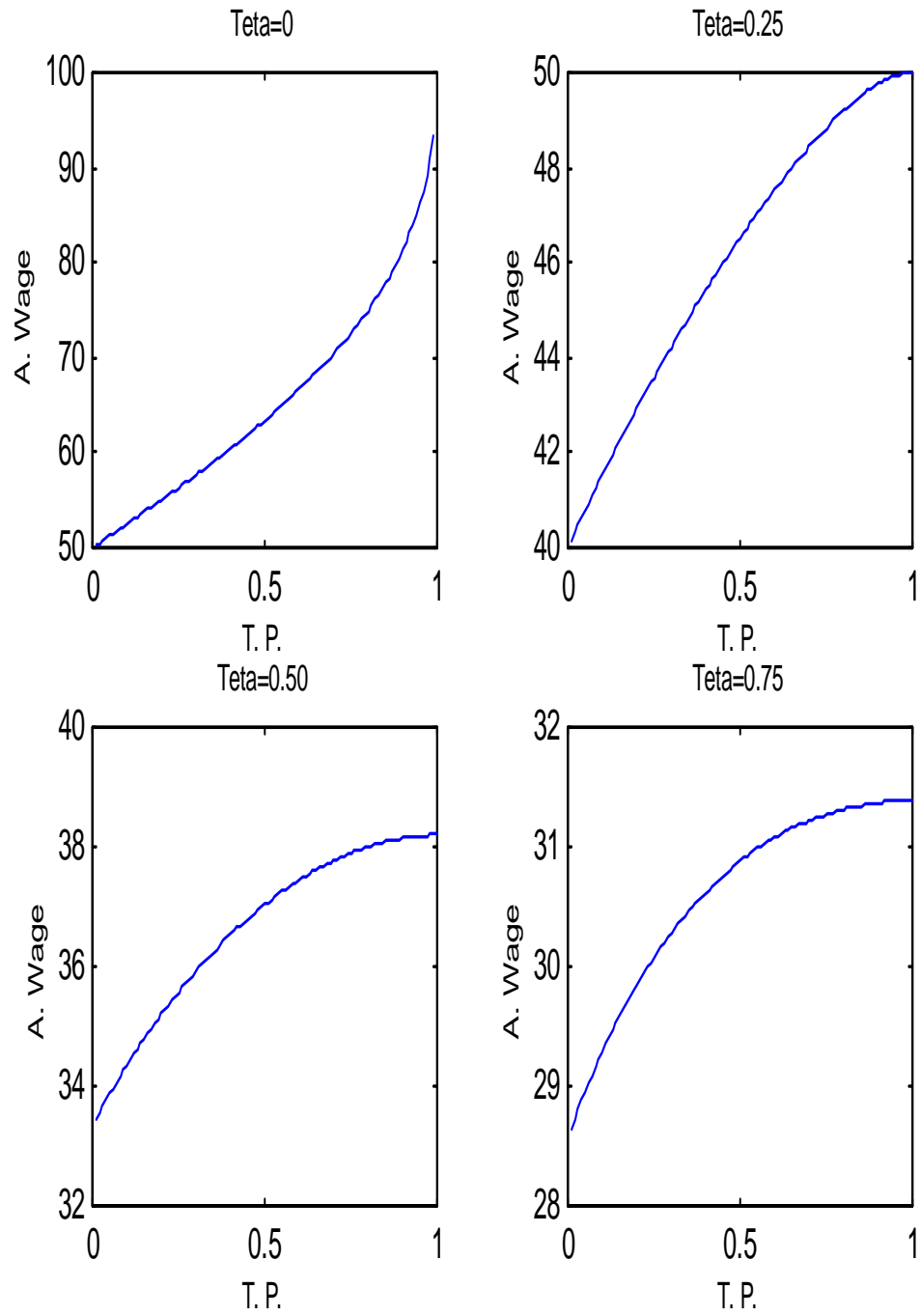


Figure 4: (Example 2) - Average Wage as a Function of Time Preference For Different Values of Theta and $T=1$.²⁸

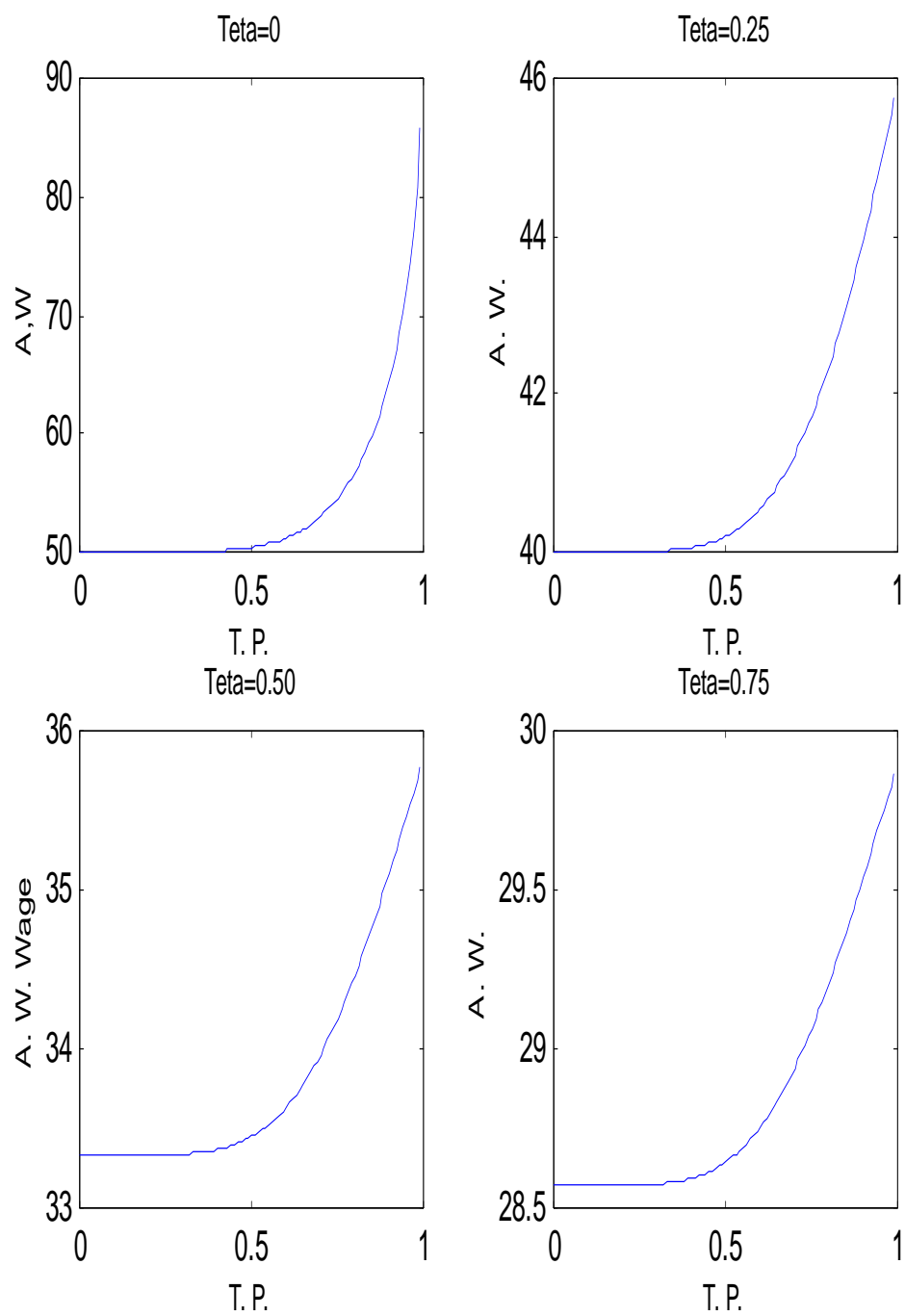


Figure 5: (Example 2) – Average Wage as a Function of Time Preference for Different Values of Theta and $T=6$. ²⁹

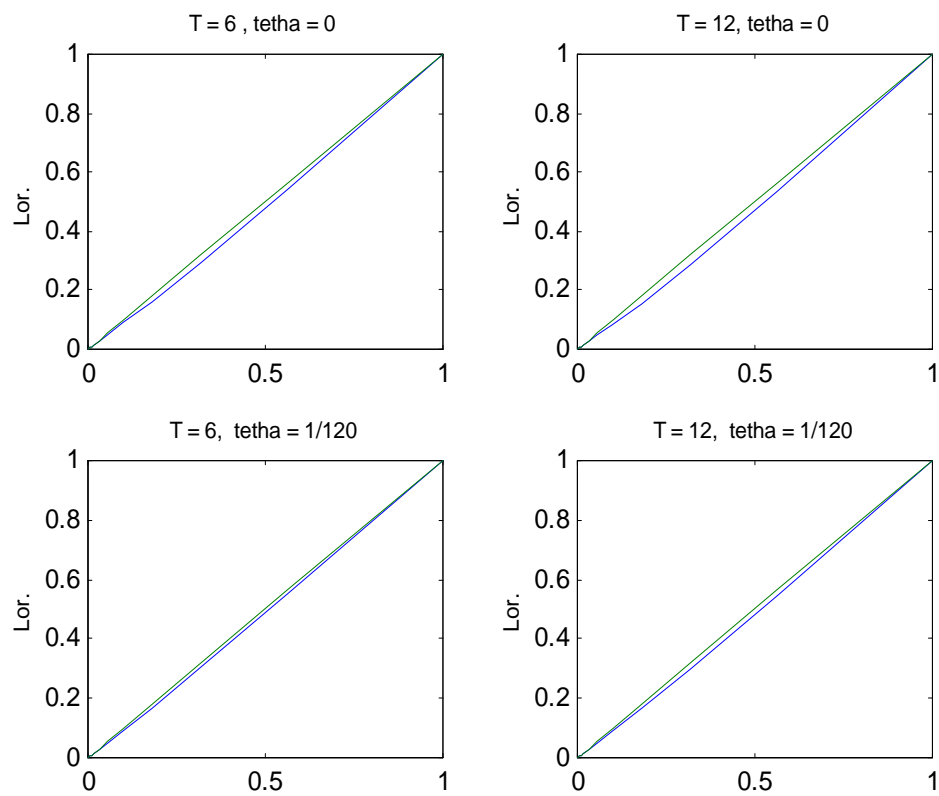


Figure 6: Between-Cohort Lorenz Curves for Different Values of T an Theta.

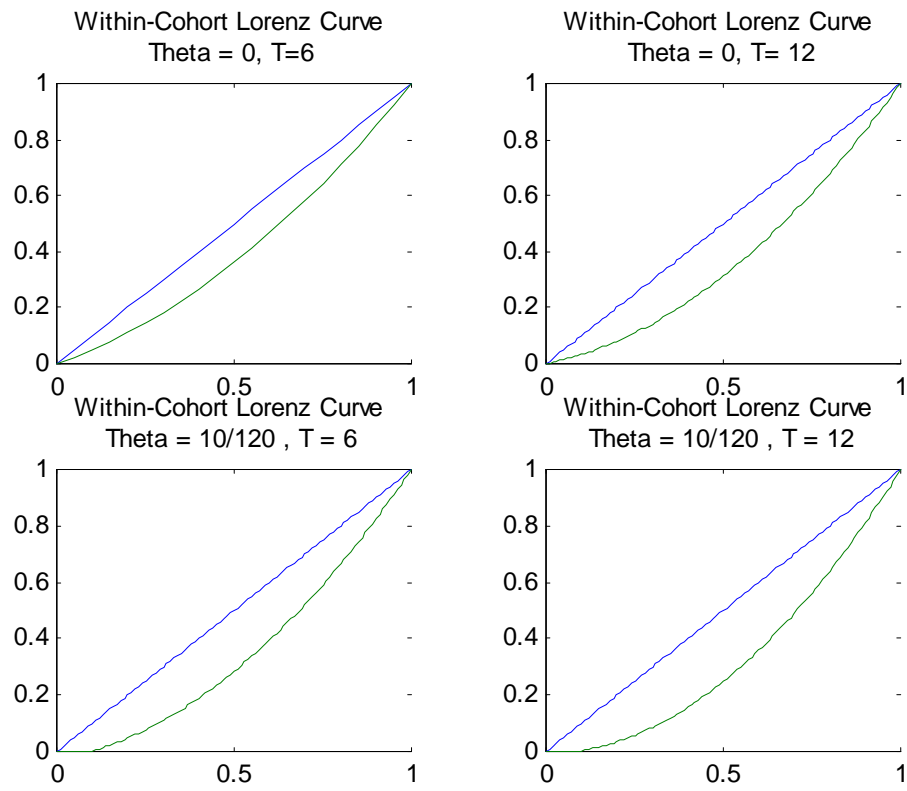


Figure 7: Within-Cohort Lorenz Curves for Different Values of T and Theta

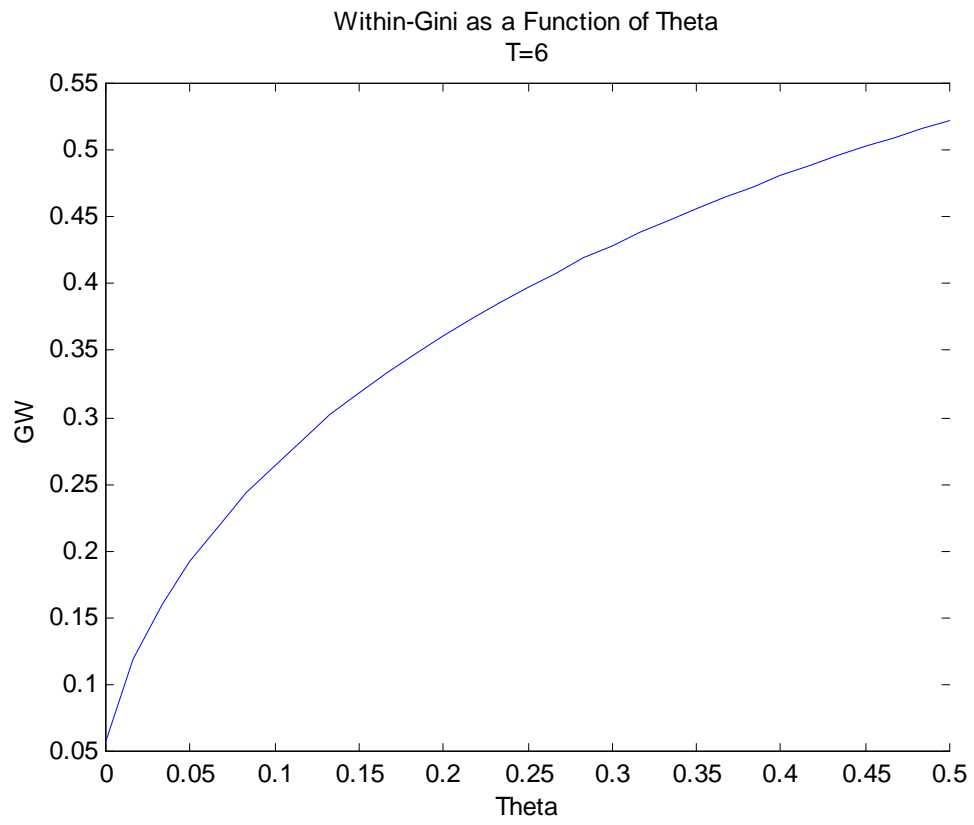


Figure 8: Average Within-Cohort Gini Coefficient (GW) as a Function of Theta, T=6

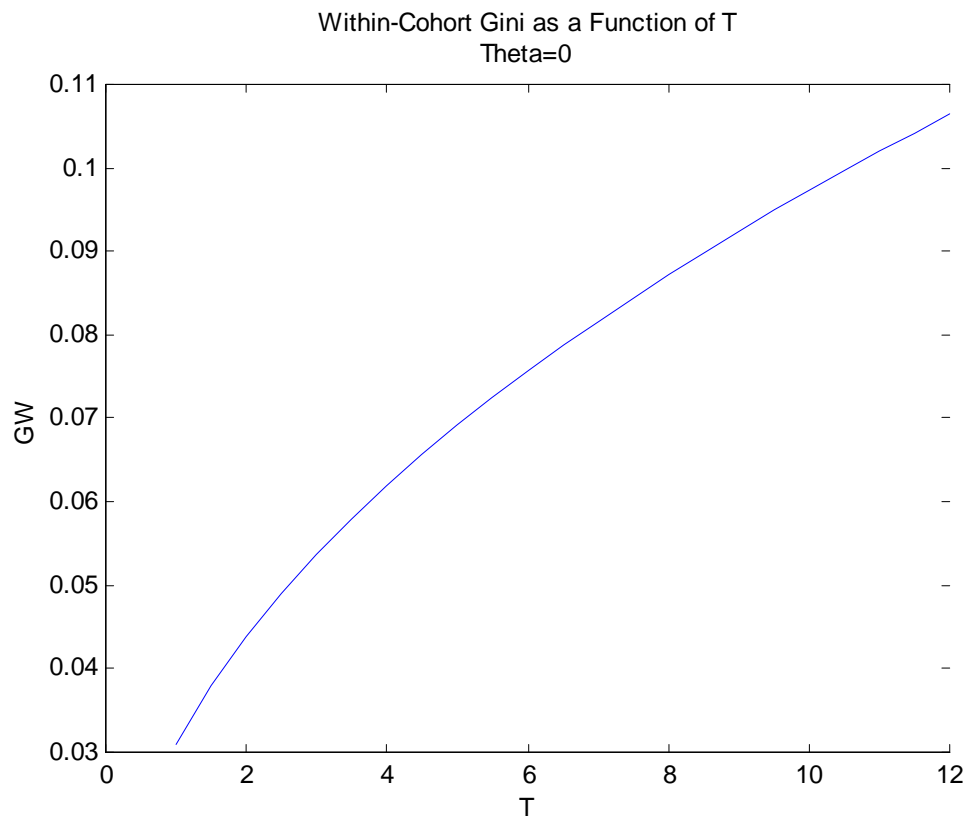


Figure 9: Average Within-Cohort Gini Coefficient (GW) as a function of T (Theta =0)

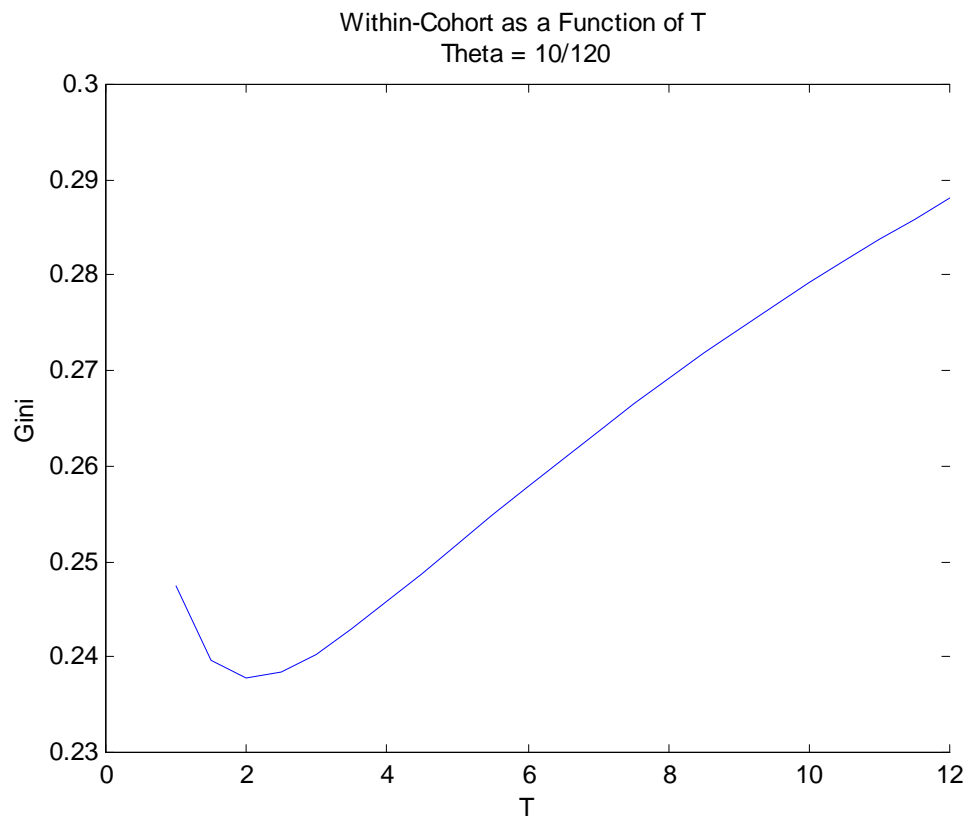


Figure 10: Average Within-Cohort Gini Coefficient (GW) as a function of T (Theta =10/120)

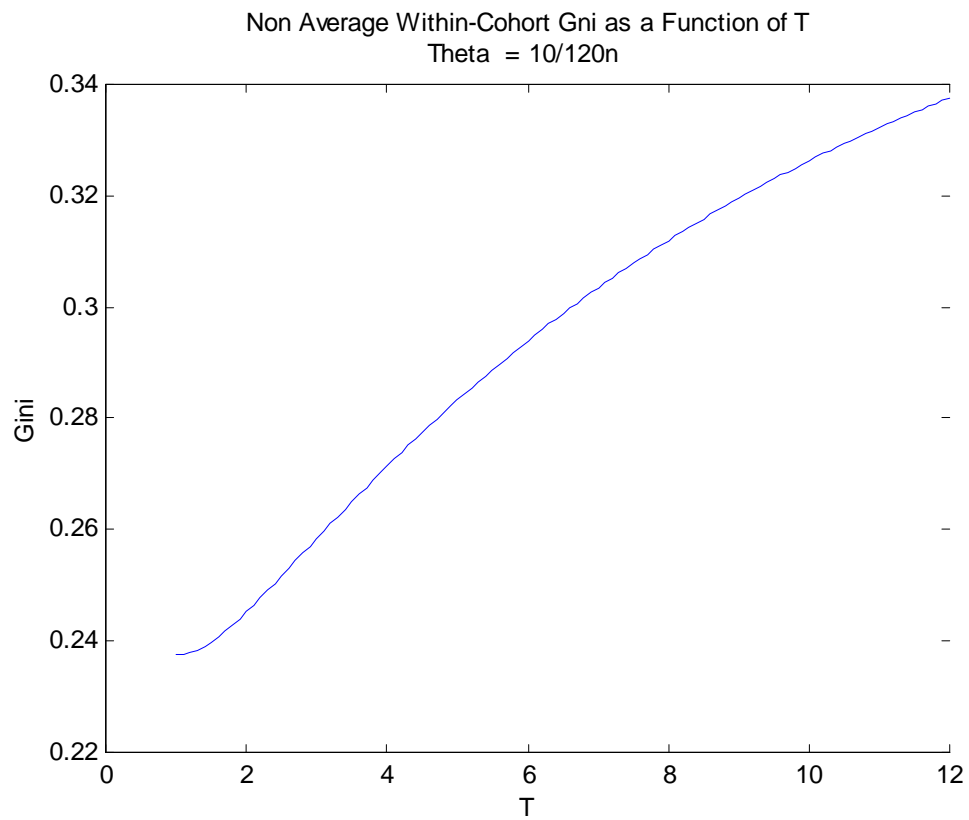


Figure 11: Non-Averaged Within-Cohort Gini Coefficient (GW) as a function of T (Theta =10/120), Cohort With Yearly Time Preference 0.9.

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